(*) If the tangent at $(x_1, y_1)$ to the curve $x^3+y^3=a^3$ meets the curve again in $(x_2, y_2)$ show that

$$\frac{x_2}{x_1}+\frac{y_2}{y_1}=-1.$$

Ans, Given equation of the curve

$$x^3+y^3=a^3$$

D.C. w.r.to $x$

$$3x^2+3y^2\frac{dy}{dx}=0$$

or $$\frac{dy}{dx}=\frac{-3x^2}{3y^2}=\frac{-x^2}{y^2}$$

104 At a point $(x_1, y_1)$ Dc-104

$$\frac{dy}{dx} = \frac{-x_1^2}{y_1^2}$$

$\therefore$ The Equation of tangent is

$$Y - y = \frac{dy}{dx}(X - x)$$

So, Equation of tangent at $(x_1, y_1)$ is

$$y - y_1 = (-)\frac{x_1^2}{y_1^2}(x - x_1)$$

or $y y_1^2 - y_1^3 = -x \cdot x_1^2 + x_1^3$

or $x \cdot x_1^2 + y \cdot y_1^2 = x_1^3 + y_1^3$

~~Since point $(x_1, y_1)$ lies on the curve~~
so the equation of curve is

$$x_1^3 + y_1^3 = a^3 \quad \ldots (2)$$

So, $x \cdot x_1^2 + y \cdot y_1^2 = a^3 \quad \ldots (3)$

Given that the tangent at $(x_1, y_1)$ to the curve also passes through $(x_2, y_2)$ so it will satisfy the equation (3)

$$x_2 \cdot x_1^2 + y_2 \cdot y_1^2 = a^3 \quad \ldots (4)$$

And also the equation of curve will be

$$x_2^3 + y_2^3 = a^3 \quad \ldots (5)$$

equating Equation (2) and (4)

$$x_2 \cdot x_1^2 + y_2 \cdot y_1^2 = x_1^3 + y_1^3$$

or $x_2 x_1^2 - x_1^3 = y_1^3 - y_2 \cdot y_1^2$

or $x_1^2(x_2 - x_1) = y_1^2(y_1 - y_2)$

or $x_1^2(x_1 - x_2) = y_1^2(y_2 - y_1) \quad \ldots (6)$

Again from equation (4) & (5) 

$$x_2 \cdot x_1^2 + y_2 \cdot y_1^2 = x_2^3 + y_2^3$$

or $x_2 \cdot x_1^2 - x_2^3 = y_2^3 - y_2 \cdot y_1^2$

or $x_2 (x_1^2 - x_2^2) = y_2 (y_2 - y_1^2)$

or $x_2 (x_1 + x_2)(x_1 - x_2) = y_2 (y_2 + y_1)(y_2 - y_1) \quad \text{--- (7)}$

Dividing equation (6) by equation (7)

$$\frac{x_1^2 \cancel{(x_1 - x_2)}}{x_2 (x_1 + x_2) \cancel{(x_1 - x_2)}} = \frac{y_1^2 \cancel{(y_2 - y_1)}}{y_2 (y_2 + y_1) \cancel{(y_2 - y_1)}}$$

or $\dfrac{x_1^2}{x_2 (x_1 + x_2)} = \dfrac{y_1^2}{y_2 (y_1 + y_2)}$

or $\dfrac{x_2 (x_1 + x_2)}{x_1^2} = \dfrac{y_2 (y_1 + y_2)}{y_1^2}$

or $\dfrac{x_1 x_2 + x_2^2}{x_1^2} = \dfrac{y_1 y_2 + y_2^2}{y_1^2}$

or $\dfrac{\cancel{x_1} x_2}{x_1^{\cancel{2}}} + \dfrac{x_2^2}{x_1^2} = \dfrac{\cancel{y_1} y_2}{y_1^{\cancel{2}}} + \dfrac{y_2^2}{y_1^2}$

or $\dfrac{x_2}{x_1} + \dfrac{x_2^2}{x_1^2} = \dfrac{y_2}{y_1} + \dfrac{y_2^2}{y_1^2}$

or $\dfrac{x_2}{x_1} - \dfrac{y_2}{y_1} = \dfrac{y_2^2}{y_1^2} - \dfrac{x_2^2}{x_1^2} = \left(\dfrac{y_2}{y_1} + \dfrac{x_2}{x_1}\right)\left(\dfrac{y_2}{y_1} - \dfrac{x_2}{x_1}\right)$

or $-\cancel{\left(\dfrac{y_2}{y_1} - \dfrac{x_2}{x_1}\right)} = \left(\dfrac{y_2}{y_1} + \dfrac{x_2}{x_1}\right)\cancel{\left(\dfrac{y_2}{y_1} - \dfrac{x_2}{x_1}\right)}$

or $\dfrac{x_2}{x_1} + \dfrac{y_2}{y_1} = -1$ Proved

106 (*) Find the Condition that D.C./06.
$x\cos\alpha + y\sin\alpha = p$ may be a tangent to the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Ans → Given equation of the curve is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \cdots (1)$

D.C. w.r.t. to $x$

$$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0$$

or $\frac{2y}{b^2}\frac{dy}{dx}=\frac{-2x}{a^2}$

or $\frac{dy}{dx}=(-)\frac{b^2}{a^2}\cdot\frac{x}{y}$

Again at the point $(x_1,y_1)$ of the curve

$$\frac{dy}{dx}=-\frac{b^2}{a^2}\frac{x_1}{y_1}\cdots (2)$$

Again equation of curve at point $(x_1,y_1)$ is

$$\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}=1\cdots (3)$$

Now equation of the tangent to the curve at the point $(x_1,y_1)$ is

$$y-y_1=\frac{dy}{dx}(x-x_1)$$

$$=(-)\frac{b^2}{a^2}\frac{x_1}{y_1}(x-x_1)\ [\text{from eqn } (2)]$$

or $(y-y_1)\frac{y_1}{b^2}+(x-x_1)\frac{x_1}{a^2}=0\cdots (4)$

109

DC-109

or $\frac{yy_1}{b^2} - \frac{y_1^2}{b^2} + \frac{xx_1}{a^2} - \frac{x_1^2}{a^2} = 0$

or $\frac{yy_1}{b^2} + \frac{xx_1}{a^2} = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 1 \ldots (5)$ [by equ (3)]

Given that the tangent to the curve is $x\cos\alpha + y\sin\alpha = p$

or $\frac{x}{p}\cos\alpha + \frac{y}{p}\sin\alpha = 1 \ldots (6)$

Comparing equ$^n$ (5) and (6) we get

$$\frac{x_1}{a^2} = \frac{\cos\alpha}{p} \text{ and } \frac{y_1}{b^2} = \frac{\sin\alpha}{p}$$

or $x_1 = \frac{a^2\cos\alpha}{p}$, $y_1 = \frac{b^2\sin\alpha}{p}$

Putting the above values in equ$^n$ (3)

$$\frac{\frac{a^4\cos^2\alpha}{p^2}}{\cancel{a^2}} + \frac{\frac{b^4\sin^2\alpha}{p^2}}{b^2} = 1$$

or $\frac{a^2\cos^2\alpha}{p^2} + \frac{b^2\sin^2\alpha}{p^2} = 1$

or $a^2\cos^2\alpha + b^2\sin^2\alpha = p^2$

This is the required condition.

108 (4) Prove that the condition that $x\cos\alpha + y\sin\alpha = p$ should touch $x^m y^n = a^{m+n}$ is DC-108

$$p^{m+n} \cdot m^m \cdot n^n = (m+n)^{m+n} \cdot a^{m+n} \sin^n\alpha \cos^m\alpha.$$

Ans → we know that the equation of the tangent at a point $(x,y)$ is

$$Y - y = \frac{dy}{dx}(X - x) \quad \ldots\ldots (1)$$

Given that

$$x^m y^n = a^{m+n}$$

Taking logarithm

$$\log x^m + \log y^n = \log a^{(m+n)}$$

or $m\log x + n\log y = (m+n)\log a$

D.c. w.r to x

$$m\cdot\frac{1}{x} + n\cdot\frac{1}{y}\frac{dy}{dx} = 0$$

or $\frac{n}{y}\cdot\frac{dy}{dx} = -\frac{m}{x}$

or $\frac{dy}{dx} = -\frac{m}{n}\cdot\frac{y}{x} \quad \ldots (2)$

From equation (1) $Y - y = (-)\frac{m}{n}\frac{y}{x}(X - x)$

or $Y\cdot nx - y\cdot nx = -my\cdot X + my\cdot x$

or $myX + nx\cdot Y = m\cdot xy + n\cdot xy$

or $my\cdot X + nx\cdot Y = (m+n)xy \quad \ldots (3)$

Now if $X\cos\alpha + Y\sin\alpha = p \ldots (4)$ be the tangent to the curve then

109 equation ③ and equation ① may be compared because they are identical. DC-109

$$\frac{my}{\cos\alpha} = \frac{nx}{\sin a} = \frac{(m+n)xy}{p}$$

From $\frac{m\cancel{y}}{\cos\alpha} = \frac{(m+n)x\cancel{y}}{p}$

or $x = \frac{m \cdot p}{(m+n)\cos\alpha}$

from $\frac{n\cancel{x}}{\sin\alpha} = \frac{(m+n)\cancel{x}y}{p}$

or $y = \frac{n \cdot p}{(m+n)\sin a}$

Putting the values of $x$ and $y$ in $x^m \cdot y^n = a^{m+n}$ we get

$$\frac{m^m \cdot p^m}{(m+n)^m \cos^m\alpha} \cdot \frac{n^n \cdot p^n}{(m+n)^n \sin^n a} = a^{m+n}$$

or $p^{m+n} \cdot m^m \cdot n^n = (m+n)^{m+n} \cdot a^{m+n} \cdot \sin^n\alpha \cdot \cos^m\alpha$

④ Find the equation of the tangent ~~to~~ the curve $y = be^{-\frac{x}{a}}$ at the point where the curve crosses the axis of $y$.

or

Prove that $\frac{x}{a} + \frac{y}{b} = 1$ touches the curve $y = be^{-\frac{x}{a}}$ at the point where the curve crosses the axis of $y$.

110 Ans → we know that 

So the curve crosses the axis at the point $(0, b)$. equation of y-axis is $x = 0$

So the equation of tangent is

$$Y - y = (X - x)\frac{dy}{dx} \quad \cdots (1)$$

Given that

$$y = be^{-\frac{x}{a}}$$

D.C. w.r.t $x$

$$\frac{dy}{dx} = b\cdot(-)\frac{1}{a}e^{-\frac{x}{a}} = -\frac{b}{a}e^{-\frac{x}{a}}$$

when $x = 0$ and $y = b$

then $\frac{dy}{dx} = -\frac{b}{a}e^{0} = -\frac{b}{a}$

or $\frac{dy}{dx} = -\frac{b}{a}$

So equation (1) becomes

$$Y - b = (X - 0)\left(-\frac{b}{a}\right)$$

or $Y - b = -\frac{b}{a}\cdot X$

or $\frac{Y - b}{b} = \frac{-X}{a}$

or $\frac{Y}{b} - \frac{b}{b} = \frac{-X}{a}$

or $\frac{X}{a} + \frac{Y}{b} = 1$

If for the current coordinates on the tangent we take $(x, y)$ instead of $(X, Y)$ the equation is

$$\frac{x}{a} + \frac{y}{b} = 1$$

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111 DC-111

(8) Find the normal to $\sqrt{xy} = a+x$, which makes equal intercepts on the axes.

Ans→ The equation of the curve is

$$\sqrt{xy} = a+x$$

or $xy = (a+x)^2 = a^2+2ax+x^2$

or $y = \frac{a^2}{x} + 2a + x$

D.C. w.r. to $x$

$$\frac{dy}{dx} = \frac{-a^2}{x^2} + 0 + 1 = \frac{x^2-a^2}{x^2} \quad \ldots (1)$$

we know that $\frac{dx}{dy} = \pm 1$

[where the normal to the curve makes equal intercepts]

or from (1) $\Rightarrow \frac{dx}{dy} = \frac{x^2}{x^2-a^2}$

So, Now $\frac{x^2}{x^2-a^2} = \pm 1$

or $x^2 = \pm (x^2-a^2)$

Taking (+) sign gives absurd result.

So Taking (−) sign

$$x^2 = -(x^2-a^2) = -x^2+a^2$$

or $2x^2 = a^2$

or $x^2 = \frac{a^2}{2} \Rightarrow x = \pm \frac{a}{\sqrt{2}}$

Hence at the points $x = \pm\frac{a}{\sqrt{2}}$, the normal of the curve makes equal intercepts on the axes.

112 ✪ If the normal at any point D.C-112 to the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ makes angle $\phi$ with the axis show that its equation is

$$y\cos\phi - x\sin\phi = a\cos 2\phi.$$

Ans → Equation of the curve is

$$f(x,y) = x^{\frac{2}{3}}+y^{\frac{2}{3}}-a^{\frac{2}{3}}=0 \cdots (1)$$

We know that Equation of the normal at point $(x,y)$ is

$$\frac{X-x}{f_x} = \frac{Y-y}{f_y}, \text{ where}$$

$(X,Y)$ are the current coordinates on the tangents

∴ D.C. equn (1) w.r to $x$

$$f_x = \frac{2}{3}x^{\frac{2}{3}-1} = \frac{2}{3}x^{-\frac{1}{3}}$$

D.C. equation (1) w.r to $y$

$$f_y = \frac{2}{3}y^{-\frac{1}{3}}$$

Now →
$$\frac{X-x}{\cancel{\frac{2}{3}}x^{-\frac{1}{3}}} = \frac{Y-y}{\cancel{\frac{2}{3}}y^{-\frac{1}{3}}}$$

or $x^{\frac{1}{3}}(X-x) = y^{\frac{1}{3}}(Y-y) \cdots (1)$

Given that normal makes angle $\phi$ with the axis

∴ m of the normal $= \tan\phi = \dfrac{x^{\frac{1}{3}}}{y^{\frac{1}{3}}}$

113 DC.113

or $\dfrac{\sin\phi}{\cos\phi} = \dfrac{x^{\frac{1}{3}}}{y^{\frac{1}{3}}}$

or $\dfrac{\sin\phi}{x^{\frac{1}{3}}} = \dfrac{\cos\phi}{y^{\frac{1}{3}}}$

or $\dfrac{\sin\phi}{x^{\frac{1}{3}}} = \dfrac{\cos\phi}{y^{\frac{1}{3}}}$

or $x^{\frac{1}{3}} = a^{\frac{1}{3}}\sin\phi$ & $y^{\frac{1}{3}} = a^{\frac{1}{3}}\cos\phi$

or $x = a\sin^3\phi$ & $y = a\cos^3\phi$

From equ$^n$ (1)

$\cancel{a^{\frac{1}{3}}}\sin\phi\,(X - a\sin^3\phi) = \cancel{a^{\frac{1}{3}}}\cos\phi\,(Y - a\cos^3\phi)$

or $X\sin\phi - a\sin^4\phi = Y\cos\phi - a\cos^4\phi$

or $X\sin\phi - Y\cos\phi = a\sin^4\phi - a\cos^4\phi = a(\sin^4\phi - \cos^4\phi)$

or $X\sin\phi - Y\cos\phi = a(\sin^2\phi + \cos^2\phi)(\sin^2\phi - \cos^2\phi)$

$= a\cdot 1\cdot(\sin^2\phi - \cos^2\phi)$

$= a(-)\cos 2\phi$

or $Y\cos\phi - X\sin\phi = a\cos 2\phi$

(*) Find the condition that the Conics $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ shall cut orthogonally.

Ans → The equation of the given Conics are

$f(x,y) = ax^2 + by^2 - 1 = 0 \quad \cdots (1)$

$F(x,y) = a_1x^2 + b_1y^2 - 1 = 0 \quad \cdots (2)$

114

DC-114

We know that if two conics cut eachother orthogonally at a point $(x, y)$ then

$$F_x f_x + F_y f_y = 0 \quad \ldots\ldots (3)$$

D.C. (1) w.r. to $x$ and $y$ respectively,

$$f_x = 2ax, \quad f_y = 2by$$

D.C. (2) w.r. to $x$ and $y$ respectively,

$$F_x = 2a_1x, \quad F_y = 2b_1y$$

Now from equation (3)

$$2a_1x \cdot 2ax + 2b_1y \cdot 2by = 0$$

or $4aa_1x^2 + 4bb_1y^2 = 0$

or $aa_1x^2 + bb_1y^2 = 0 \quad \ldots\ldots (4)$

Equation (2) − Equation (1)

$\Rightarrow (a-a_1)x^2 + (b-b_1)y^2 = 0$

or $(a-a_1)x^2 = -(b-b_1)y^2 \quad \ldots\ldots (5)$

from (4) $\Rightarrow aa_1x^2 = -bb_1y^2 \quad \ldots\ldots (6)$

Dividing equn (5) by equn (6)

$$\frac{(a-a_1)x^2}{aa_1x^2} = \frac{(-)(b-b_1)y^2}{(-)bb_1y^2}$$

or $\dfrac{a-a_1}{aa_1} = \dfrac{b-b_1}{bb_1}$

or $\dfrac{a}{aa_1} - \dfrac{a_1}{aa_1} = \dfrac{b}{bb_1} - \dfrac{b_1}{bb_1}$

or $\dfrac{1}{a_1} - \dfrac{1}{a} = \dfrac{1}{b_1} - \dfrac{1}{b}$

or $\dfrac{1}{a} - \dfrac{1}{b} = \dfrac{1}{a_1} - \dfrac{1}{b_1}$ This is the required equation.

115

D.C.-115

(*) In the catenary $y = a\cosh\frac{x}{a}$ prove that the length of the portion of the normal intercepted between the curve and the x-axis varies as $y^2$.

**Ans,** Equation of the catenary

$$y = a\cosh\frac{x}{a}$$

D.C. w.r to x

$$\frac{dy}{dx} = \cancel{a}\sinh\frac{x}{a}\cdot\frac{1}{\cancel{a}} = \sinh\frac{x}{a}$$

$$\text{Length of the normal} = y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$= y\sqrt{1+\sinh^2\frac{x}{a}}$$

$$= y\sqrt{\cosh^2\frac{x}{a}} = y\cosh\frac{x}{a}$$

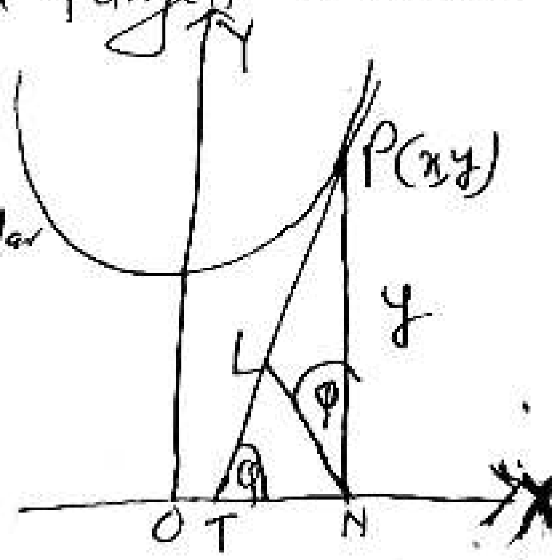
$$= y\cdot\frac{y}{a} = \frac{y^2}{a}$$

$\therefore$ Length of the normal $\propto y^2$.

(*) Prove that the catenary $y = c\cosh\frac{x}{c}$ the length of the perpendicular from the foot of the ordinate on the tangent is constant.

**Ans,** Let $P(x,y)$ be any point on the catenary. From P, Draw a perpendicular PN to x-axis. $PN = y$

Tangent PT meets x-axis at T. PT makes an angle $\phi$ with x-axis. $\therefore \angle PTN = \phi$

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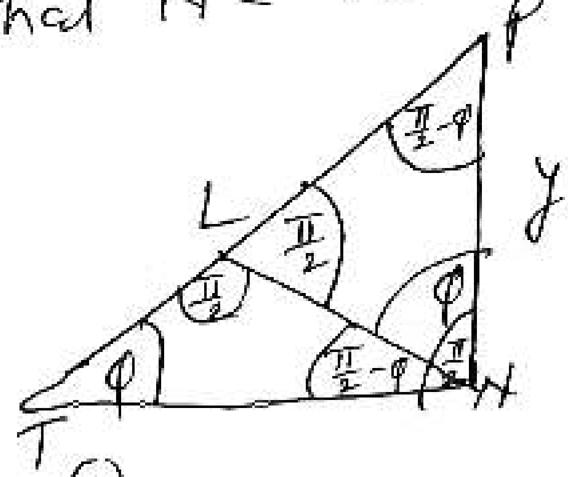
116

D.C-116

From N, Draw a perpendicular NL to PT.

we have to prove that NL is constant.




$\angle TPN = \frac{\pi}{2} - \varphi$

in $\triangle NLP$

$$\sin\left(\frac{\pi}{2} - \varphi\right) = \frac{NL}{PN}$$

or $$\cos\varphi = \frac{NL}{y}$$

or $$NL = y\cos\varphi \quad \ldots\ldots (1)$$

Given equation,

$$y = c\cosh\frac{x}{c}$$

D.C. w.r.to x

$$\frac{dy}{dx} = \cancel{c} \cdot \sinh\frac{x}{c} \cdot \frac{1}{\cancel{c}} = \sinh\frac{x}{c}$$

$\therefore$ $$\tan\varphi = \frac{dy}{dx}$$

or $$\tan\varphi = \sinh\frac{x}{c}$$


$\therefore$ $$\cos\varphi = \frac{1}{\sqrt{1+\sinh^2\frac{x}{c}}} = \frac{1}{\sqrt{\cosh^2\frac{x}{c}}} = \frac{1}{\cosh\frac{x}{c}}$$

or $$\cos\varphi = \frac{1}{\frac{y}{c}} = \frac{c}{y}$$

$\therefore$ $$NL = y\cos\varphi \quad \text{[from equ}^n\text{ (1)]}$$

or $$NL = \cancel{y} \cdot \frac{c}{\cancel{y}} = c$$

$\therefore$ NL = Constant

Proved

117 (*) Show that in the curve $y = be^{-\frac{a}{x}}$, the subtangent varies as the square of the abscissa. D.C-117

Ans, Equation of the curve is

$$y = be^{-\frac{a}{x}} \quad \ldots (1)$$

D.C. w.r. to $x$

$$\frac{dy}{dx} = be^{-\frac{a}{x}} \cdot (-a) \cdot \frac{(-1)}{x^2} = be^{-\frac{a}{x}} \cdot \frac{a}{x^2}$$

or $$\frac{dy}{dx} = \frac{a \cdot y}{x^2}$$

We know that

$$\text{subtangent} = \frac{y}{\frac{dy}{dx}} = \frac{y}{\frac{a \cdot y}{x^2}}$$

$$= \frac{x^2}{a}$$

Hence the subtangent varies as the square of the abscissa.

(*) Show that in the curve $y = be^{\frac{x}{a}}$ the subtangent is of constant length and the subnormal varies as the square of the ordinate.

Ans, The given equation of the curve is

$$y = be^{\frac{x}{a}} \quad \ldots (1)$$

D.C. w.r. to $x$

$$\frac{dy}{dx} = b \cdot \frac{1}{a} e^{\frac{x}{a}} = \frac{b}{a} e^{\frac{x}{a}} = \frac{y}{a}$$

or $y_1 = \frac{y}{a}$

we know that

the subtangent $= \frac{y}{y_1}$

$= \frac{y}{\frac{y}{a}} = a =$ constant

we know that

the subnormal $= y y_1$

$= y \cdot \frac{y}{a} = \frac{y^2}{a}$

Hence the subnormal varies as the square of the ordinate.

(4) show that in the curve $s = c \log \frac{c}{y}$, the tangent is of the constant length.

Ans → The equation of the curve is

$s = c \log \frac{c}{y} = c \log c - c \log y$

D.C. w.r.to y

$\frac{ds}{dy} = -c \cdot \frac{1}{y} = \frac{-c}{y}$

The length of the tangent [we know that the length of tangent $= y \operatorname{cosec}\psi$]

$= y \operatorname{cosec}\psi$

$= y \cdot \frac{ds}{dy}$

$= \cancel{y} \cdot \frac{-c}{\cancel{y}} = -c$, which is constant.

119 In the catenary $y = a\cosh\frac{x}{a}$ (DC-119) prove that the length of the portion of the normal intercepted between the curve and the x-axis varies as $y^2$.

Ans, Given that

$$y = a\cosh\frac{x}{a}$$

D.C. w.r to x

$$\frac{dy}{dx} = \cancel{a}\sinh\frac{x}{a}\cdot\frac{1}{\cancel{a}} = \sinh\frac{x}{a}$$

we know that the length of the normal $= y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

$$= a\cosh\frac{x}{a}\sqrt{1+\sinh^2\frac{x}{a}}$$

$$= a\cosh\frac{x}{a}\cdot\sqrt{\cosh^2\frac{x}{a}}$$

$$= a\cosh\frac{x}{a}\cdot\cosh\frac{x}{a}$$

$$= y\cdot\frac{y}{a} = \frac{y^2}{a}$$

So the length of the normal $\propto y^2$.

Find the angle of intersection of the cardioids

$$r = a(1+\cos\theta)$$

$$r = b(1-\cos\theta)$$

Ans → Given that the equation of first curve is $r = a(1+\cos\theta)$

120 D.C. w.r to $\theta$ DC-120

$$\frac{dr}{d\theta} = a\cdot(-)\sin\theta$$

we know that

$$\tan\phi = r\cdot\frac{d\theta}{dr}$$

$$= a\cdot(1+\cos\theta)\cdot\frac{1}{\frac{dr}{d\theta}}$$

$$= \frac{\cancel{a}(1+\cos\theta)}{(-)\cancel{a}\sin\theta} = \frac{(-)\cancel{2}\cos^2\frac{\theta}{2}}{\cancel{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$= (-)\cot\frac{\theta}{2}$$

$$\tan\phi = \tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$$

$$\therefore \phi = \frac{\pi}{4}+\frac{\theta}{2}$$

Equation of second curve is

$$r = b(1-\cos\theta)$$

D.C. w.r to $\theta$

$$\frac{dr}{d\theta} = b\sin\theta$$

we know that $\tan\phi' = r\frac{d\theta}{dr} = r\cdot\frac{1}{\frac{dr}{d\theta}}$

or $\tan\phi' = \frac{\cancel{b}(1-\cos\theta)}{\cancel{b}\sin\theta}$

$$= \frac{\cancel{2}\sin^2\frac{\theta}{2}}{\cancel{2}\cancel{\sin\frac{\theta}{2}}\cos\frac{\theta}{2}} = \tan\frac{\theta}{2}$$

or $\phi' = \frac{\theta}{2}$

The desired angle of intersection

$$= \phi - \phi'$$

$$= \frac{\pi}{4}+\frac{\theta}{2}-\frac{\theta}{2} = \frac{\pi}{2}$$

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