9 D.C. 1

(8) If $x = \cos(\log y)$
show that
$$(1-x^2)y_2 - xy_1 = y$$

Ans, Given that
$$x = \cos(\log y)$$
or $\log y = \cos^{-1}x$

D.C. w.r.to x
$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$$
or $$\frac{y_1}{y} = \frac{-1}{\sqrt{1-x^2}}$$
Squaring
$$\frac{y_1^2}{y^2} = \frac{1}{1-x^2}$$
or $(1-x^2)y_1^2 = y^2$

D.C. w.r.to x
$$(-)2x\cdot y_1^2 + (1-x^2)\,2y_1y_2 = 2yy_1$$
or $$(1-x^2)y_2 - xy_1 = y$$

(9) If $x = \sin(\log y)$
prove that $(1-x^2)y_2 - xy_1 = y$

Ans, Given that
$$x = \sin(\log y)$$
or $\log y = \sin^{-1}x$

D.C. w.r.to x

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

or $$\frac{y_1}{y} = \frac{1}{\sqrt{1-x^2}}$$

or $$y_1\cdot\sqrt{1-x^2} = y$$

Squaring

$$y_1^2(1-x^2) = y^2$$

D.C. w.r to x

$$2y_1y_2(1-x^2) + y_1^2\cdot(-2x) = 2yy_1$$

or $$(1-x^2)y_2 - xy_1 = y$$

(4) If $y = (\tan^{-1}x)^2$ prove that

$$(x^2+1)^2\cdot y_2 + 2x(x^2+1)\cdot y_1 = 2$$

Ans, Given that

$$y = (\tan^{-1}x)^2$$

D.C. w.r. to x

$$y_1 = 2\tan^{-1}x\cdot\frac{1}{1+x^2}$$

or $$(1+x^2)y_1 = 2\tan^{-1}x$$

D.C. w.r to x

$$(1+x^2)y_2 + 2xy_1 = \frac{2}{1+x^2}$$

or $$(1+x^2)^2y_2 + 2x(1+x^2)y_1 = 2$$

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11 ✡ If $x = \cosh\left(\frac{\log y}{m}\right)$ D.C. 1)

prove that

$$(x^2-1)y_2 + xy_1 - m^2 y = 0$$

Ans → we have

$$x = \cosh\left(\frac{\log y}{m}\right)$$

or $\cosh^{-1}x = \frac{\log y}{m}$

D.C. w.r.to x

$$\frac{1}{\sqrt{x^2-1}} = \frac{1}{m} \cdot \frac{1}{y} \cdot y_1$$

or $my = y_1 \sqrt{x^2-1}$ ...(1)

D.C. w.r.to x

$$my_1 = y_1 \cdot \frac{1}{\cancel{2}\sqrt{x^2-1}} \cdot \cancel{2}x + y_2\sqrt{x^2-1}$$

or $my_1 \cdot (\sqrt{x^2-1}) = x \cdot y_1 + y_2(x^2-1)$

or $m \cdot my = x \cdot y_1 + (x^2-1) \cdot y_2$ [from (1)]

or $(x^2-1)y_2 + xy_1 - m^2 y = 0$

✡ If $y = \log\left(\frac{x}{a+bx}\right)^x$. prove that

$$x^3 \cdot y_2 = (y - xy_1)^2$$

Ans, Given that $y = \log\left(\frac{x}{a+bx}\right)^x$

or $y = x \cdot \log\left(\frac{x}{a+bx}\right)$ .....(1)

or $y = x\left[\log x - \log(a+bx)\right]$

or $y = x.\log x - x\log(a+bx)$

D.C. w.r.to x

or $y_1 = x.\frac{1}{x} + \log x - x.\frac{1}{a+bx}b - \log(a+bx)$

or $y_1 = 1 + \log x - \frac{bx}{a+bx} - \log(a+bx)$

or $y_1 = \boxed{\log x - \log(a+bx)} + 1 - \frac{bx}{a+bx}$

or $y_1 = \log\frac{x}{a+bx} + \frac{a+\cancel{bx}-\cancel{bx}}{a+bx}$

or $y_1 = \frac{y}{x} + \frac{a}{a+bx}$ ......(2) [from equ^n (1)]

D.C. w.r.to x

$y_2 = \frac{xy_1 - y}{x^2} - a.\frac{1}{(a+bx)^2}.b$

Multiplying by x^3

$x^3.y_2 = x(xy_1 - y) - \frac{ab\,x^3}{(a+bx)^2}$ ....(3)

Now from equ^n (2)

$y_1 - \frac{y}{x} = \frac{a}{a+bx}$

or $\frac{xy_1 - y}{x} = \frac{a}{a+bx}$

or $xy_1 - y = \frac{ax}{a+bx}$ ....(4)

Now putting this value in (3)

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D.C-13

$$x^3 y_2 = x \cdot \frac{ax}{a+bx} - \frac{abx^3}{(a+bx)^2}$$

$$= \frac{ax^2 \cdot (a+bx) - abx^3}{(a+bx)^2}$$

$$= \frac{ax^2 [a+bx - bx]}{(a+bx)^2}$$

$$= \frac{ax^2 \cdot a}{(a+bx)^2} = \frac{a^2 x^2}{(a+bx)^2}$$

$$= \left(\frac{ax}{a+bx}\right)^2$$

$$x^3 y_2 = (xy_1 - y)^2 \quad \text{[from equ}^n \text{ (4)]}$$

or $x^3 \cdot y_2 = (y - xy_1)^2$

(*) If $y = A(x+\sqrt{x^2-1})^n + B(x-\sqrt{x^2-1})^n$

prove that

$$(x^2-1) y_2 + x y_1 - n^2 y = 0$$

Ans) Given that

$$y = A(x+\sqrt{x^2-1})^n + B(x-\sqrt{x^2-1})^n \quad \text{......(1)}$$

D.C. w.r to x

$$y_1 = n \cdot A (x+\sqrt{x^2-1})^{n-1} \left(1 + \frac{1}{2\sqrt{x^2-1}} \cdot 2x\right)$$

$$+ nB (x-\sqrt{x^2-1})^{n-1} \left(1 - \frac{1}{2\sqrt{x^2-1}} \cdot 2x\right)$$

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or $y_1 = nA(x+\sqrt{x^2-1})^{n-1}\left(\frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}}\right)$
$+nB(x-\sqrt{x^2-1})^{n-1}\left(\frac{\sqrt{x^2-1}-x}{\sqrt{x^2-1}}\right)$

$= nA(x+\sqrt{x^2-1})^{n-1}\left(\frac{x+\sqrt{x^2-1}}{\sqrt{x^2-1}}\right)$
$-nB(x-\sqrt{x^2-1})^{n-1}\left(\frac{x-\sqrt{x^2-1}}{\sqrt{x^2-1}}\right)$

or $y_1 \cdot \sqrt{x^2-1} = nA(x+\sqrt{x^2-1})^n - nB\cdot(x-\sqrt{x^2-1})^n$

D.C. w.r.t. x

$y_2\sqrt{x^2-1} + y_1 \cdot \frac{1}{\cancel{2}\sqrt{x^2-1}}\cdot\cancel{2}x = n^2A\cdot(x+\sqrt{x^2-1})^{n-1}$
$\cdot\left(1+\frac{1}{\cancel{2}\sqrt{x^2-1}}\cancel{2}x\right) - n^2B\cdot(x-\sqrt{x^2-1})^{n-1}$
$\left(1-\frac{1}{\cancel{2}\sqrt{x^2-1}}\cdot\cancel{2}x\right)$

or $(x^2-1)y_2 + xy_1 = n^2A\cdot(x+\sqrt{x^2-1})^{n-1}\left(\frac{x+\sqrt{x^2-1}}{\cancel{\sqrt{x^2-1}}}\right)\cdot\cancel{\sqrt{x^2-1}}$
$+n^2B(x-\sqrt{x^2-1})^{n-1}\left(\frac{x-\sqrt{x^2-1}}{\cancel{\sqrt{x^2-1}}}\right)\cdot\cancel{\sqrt{x^2-1}}$

or $(x^2-1)y_2 + xy_1 = n^2A\cdot(x+\sqrt{x^2-1})^n + n^2B(x-\sqrt{x^2-1})^n$
or $(x^2-1)y_2 + xy_1 = n^2\left[A(x+\sqrt{x^2-1})^n + B(x-\sqrt{x^2-1})^n\right]$

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or $(x^2-1)y_2 + x y_1 = n^2 . y$ [from eq^n (1)] DC-15

D.C. w.r.to x it n times by

Leibnitz's theorem we get

$(x^2-1) y_{n+2} + {}^nc_1 \, y_{n+1} (2x) + {}^nc_2 \, y_n (\cdot 2) + y_{n+1}(x)$

$+ {}^nc_1 \, y_n (\cdot 1) - n^2 y_n = 0$

or $(x^2-1) y_{n+2} + \frac{n}{1} y_{n+1} \cdot 2x + \frac{n(n-1)}{1 \cdot \cancel{2}} \cdot y_n \cdot \cancel{2}$

$+ y_{n+1} \cdot x + \frac{n}{1} \cdot y_n - n^2 y_n = 0$

or $y_{n+2}(x^2-1) + y_{n+1}(2nx + x) + y_n \{n(n-1) + n - n^2\} = 0$

or $y_{n+2}(x^2-1) + y_{n+1} \cdot x (1+2n) + y_n \{\cancel{n^2} - \cancel{n} + \cancel{n} - \cancel{n^2}\} = 0$

or $(x^2-1) y_{n+2} + x(1+2n) y_{n+1} = 0$