✡ Solve $x\dfrac{d^2y}{dx^2} - 2(x+1)\dfrac{dy}{dx} + (x+2)y = (x-2)e^{2x}$

Ans → Given that

$$x\cdot\frac{d^2y}{dx^2} - 2(x+1)\frac{dy}{dx} + (x+2)y = (x-2)e^{2x}$$

or $$\frac{d^2y}{dx^2} - 2\left(\frac{x+1}{x}\right)\frac{dy}{dx} + \left(\frac{x+2}{x}\right)y = \frac{x-2}{x}e^{2x} \quad \ldots (1)$$

we know that the standard form is

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R \quad \ldots (2)$$

Scanned with CamScan

Comparing equ^n (1) & (2) we get

$$P = -\frac{2(x+1)}{x}, \quad Q = \frac{x+2}{x}, \quad R = \frac{x-2}{x}e^{2x}$$

Here we get

$$1+P+Q = 1 - \frac{2x+2}{x} + \frac{x+2}{x}$$

$$= \frac{x - 2x \cancel{-2} + x \cancel{+2}}{x} = 0$$

So, $y = e^x$ is a part of C.F. of the solution of equ^n (1)

Now put $y = v e^x$

So, $\frac{dy}{dx} = \frac{dv}{dx}e^x + v e^x$

& $\frac{d^2y}{dx^2} = \frac{d^2v}{dx^2}e^x + 2\frac{dv}{dx}e^x + v e^x$

Now, from equ^n (1)

$$e^x\left[\frac{d^2v}{dx^2} + 2\frac{dv}{dx} + v\right] - \frac{2(x+1)}{x}\left(\frac{dv}{dx} + v\right)e^x + \left(\frac{x+2}{x}\right)\cdot v e^x = \frac{x-2}{x}\cdot e^{2x}$$

or $$\frac{d^2v}{dx^2} + \frac{dv}{dx}\left(2 - \frac{2x+2}{x}\right) + v - 2\left(\frac{x+1}{x}\right)v + \left(\frac{x+2}{x}\right)\cdot v = \frac{x-2}{x}e^x$$

or $$\frac{d^2v}{dx^2} + \frac{dv}{dx}\left(\frac{\cancel{2x}-\cancel{2x}-2}{x}\right) + v\left[1 - \frac{2x+2}{x} + \frac{x+2}{x}\right] = \frac{x-2}{x}e^x$$

or $$\frac{d^2v}{dx^2} - \frac{2}{x}\frac{dv}{dx} + v\cdot 0 = \frac{x-2}{x}e^x \quad [\text{As, } 1+P+Q=0]$$

Now $p = \frac{dv}{dx}$

$$\frac{dp}{dx} = \frac{d^2v}{dx^2}$$

13 $\qquad$ 13

$$\frac{dp}{dx} - \frac{2}{x}p = \frac{x-2}{x}e^x$$

which is linear

$$I.F. = e^{(-)\int 2\frac{dx}{x}} = e^{-2\log x} = e^{\log x^{-2}} = \frac{1}{x^2}$$

$$\therefore p\cdot\frac{1}{x^2} = \int\left(\frac{x-2}{x}\right)e^x\cdot\frac{1}{x^2}\,dx + k$$

$$= \int e^x\left(\frac{1}{x^2} - \frac{2}{x^3}\right)dx + k$$

$$= \int e^x\cdot\frac{1}{x^2}\,dx - 2\int e^x\cdot\frac{1}{x^3}\,dx + k$$

$$\because \int e^x\{f(x) + f'(x)\}dx = e^x f(x)$$

$$\text{So } p\cdot\frac{1}{x^2} = e^x\cdot\frac{1}{x^2} + k_1$$

$$\text{or } \frac{dv}{dx} = e^x + k_1x^2$$

$$\text{or } dv = e^x\,dx + k_1x^2\,dx$$

Integrating

$$\int dv = \int e^x\,dx + k_1\int x^2\,dx$$

$$\text{or } v = e^x + k_1\frac{x^3}{3} + k_2$$

Now the C.S. is

$$y = ve^x = e^{2x} + \frac{1}{3}e^x\cdot x^3 + k_2e^x$$

14 ④ Solve

$$x\frac{d^2y}{dx^2} - (2x+1)\frac{dy}{dx} + (x+1)y = (x^2+x-1)e^{2x}$$ 14

Ans) Given that

$$x\cdot\frac{d^2y}{dx^2} - (2x+1)\frac{dy}{dx} + (x+1)y = (x^2+x-1)e^{2x}$$

or $$\frac{d^2y}{dx^2} - \left(\frac{2x+1}{x}\right)\frac{dy}{dx} + \left(\frac{x+1}{x}\right)y = \frac{x^2+x-1}{x}e^{2x} \quad \cdots (1)$$

Here, $P = (-)\frac{2x+1}{x}$, $Q = \frac{x+1}{x}$, $R = \frac{x^2+x-1}{x}$

where $$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

Here, $$1+P+Q = 1 - \frac{2x+1}{x} + \frac{x+1}{x}$$

$$= \frac{x - 2x \cancel{-1} + x \cancel{+1}}{x} = 0$$

$\therefore$ $y = e^x$ is a part of the C.F.

Now putting $y = ve^x$

D.C.

$$\frac{dy}{dx} = \frac{dv}{dx}e^x + ve^x$$

& $$\frac{d^2y}{dx^2} = \frac{d^2v}{dx^2}e^x + 2\frac{dv}{dx}e^x + ve^x$$

From eq^n (1)

$$e^x\left[\frac{d^2v}{dx^2} + 2\frac{dv}{dx} + v\right] - \left(\frac{2x+1}{x}\right)\left(\frac{dv}{dx} + v\right)e^x + \left(\frac{x+1}{x}\right)ve^x = \frac{x^2+x-1}{x}e^{2x}$$

or $$\frac{d^2v}{dx^2} + \frac{dv}{dx}\left[2 - \frac{2x+1}{x}\right] + \left[v - \frac{2x+1}{x}\cdot v + \frac{x+1}{x}\cdot v\right] = \frac{x^2+x-1}{x}e^x$$

or, $\frac{d^2v}{dx^2} + \frac{dv}{dx}\left[\frac{\cancel{2x}-\cancel{2x}-1}{x}\right] + v\left[1 - \frac{2x+1}{x} + \frac{x+1}{x}\right] = \frac{x^2+x-1}{x}e^x$

or $\frac{d^2v}{dx^2} - \frac{1}{x}\frac{dv}{dx} + v \cdot 0 = \frac{x^2+x-1}{x}e^x$ $\left[\text{As, } 1+P+Q=0\right]$

Now $p = \frac{dv}{dx}$

$\frac{dp}{dx} = \frac{d^2v}{dx^2}$

or $\frac{dp}{dx} - \frac{1}{x} \cdot p = \frac{x^2+x-1}{x}e^x$

which is linear

$I \cdot F = e^{(-)\int \frac{1}{x}dx} = e^{-\log x} = e^{\log x^{-1}} = \frac{1}{x}$

$\therefore p \cdot \frac{1}{x} = \int \frac{1}{x} \cdot \frac{x^2+x-1}{x} e^x \, dx + k$

$= \int e^x \left[\frac{x^2}{x^2} + \frac{x}{x^2} - \frac{1}{x^2}\right] dx + k$

$= \int e^x \left[\left(1+\frac{1}{x}\right) - \frac{1}{x^2}\right] dx + k$

or $p \cdot \frac{1}{x} = e^x \cdot \left(\frac{1}{x}+1\right) + k_1$

(Because $\int e^x [f(x) + f'(x)] dx = e^x f(x)$

Here $f(x) = \frac{1}{x} + 1$

$\therefore f'(x) = \frac{-1}{x^2}$)

$p = \frac{dv}{dx} = \cancel{x} \cdot e^x \left(\frac{1+x}{\cancel{x}}\right) + xk_1 = e^x(1+x) + x \cdot k_1$

integrating

$\int dv = \int x \cdot e^x dx + \int e^x dx + k_1 \int x \, dx$

16 $v = x\int e^x dx - \int\left\{\frac{d}{dx}x\int e^x dx\right\}dx + e^x + k_1 \cdot \frac{x^2}{2}$ 16

$= xe^x - \cancel{e^x} + \cancel{e^x} + k_1 \cdot \frac{x^2}{2} + k_2$

$v = xe^x + k_1 \frac{x^2}{2} + k_2$

Hence the complete solution of the given differential equation is

$y = ve^x$

$= \left(xe^x + k_1 \frac{x^2}{2} + k_2\right)e^x$

or $y = xe^{2x} + k_1 \frac{x^2}{2} e^x + k_2 e^x$

Scanned with CamScan